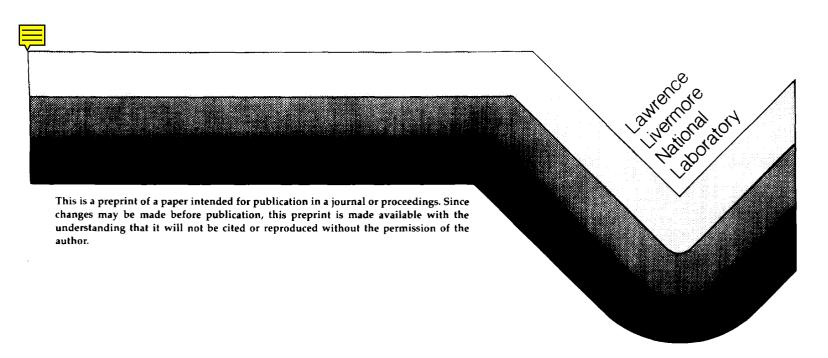Plasma Heating in a Tandem Mirror
By ICRH on High-Z Minority Ions

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This paper was prepared for submittal to
Journal of Nuclear Fusion

February 5, 1986

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PLASMA HEATING IN A TANDEM MIRROR BY ICRH ON HIGH-Z MINORITY IONS*

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ABSTRACT

The use of neutral-beam-injected high-Z ions as a minority-ion intermediary for the efficient transfer of r.f. heating energy to majority hydrogenic ions is discussed. It is shown that each such stripped high-Z minority ion of charge number q and atomic weight A_q is roughly $q^3 A_q^{1/2}$ times more effective for transferring r.f. energy than minority protons. Other attributes and advantages of the proposal are also discussed.

This note discusses the potential advantages (for plasma heating) of combining, (a) the introduction, through neutral beam injection, of a small component of energetic high-Z ions into a tandem mirror plasma with, (b) ion cyclotron resonant heating of these ions. These advantages accrue from two circumstances: (1) the r.f. coupling per ion is stronger, by a factor q, for high-Z ions stripped to the qth ionization state than it is for hydrogenic ions, and (2) collisional coupling between the heated minority high-Z ions of atomic weight A_q and the hydrogenic ions of the bulk plasma is higher, by factors of order $A_q^{1/2} q^2$ than is the case for r.f.-heated energetic hydrogenic ions. The upshot is that a very small fractional constituent of neutral beam-injected high-Z ions can act as an intermediary for the efficient heating of the ions and the electrons of the bulk plasma in a tandem mirror system. A further benefit is that the substantial distortions of the hydrogenic ion distribution function away from a Maxwellian that may tend to occur with direct ICRH of those ions should not occur when the coupling of energy to these ions arises instead through collisional encounters with intermediary high-Z ions. The elimination or reduction of such distortions should have a salutary effect on the microstability and on radial transport of the hydrogenic component of the plasma.

Minority-species ICRH has been discussed in previous articles [1] [2] [3] [4]. These papers, however, dealt mainly with the cases either of the heating of minority protons or of stripped helium ions existing as minority species in a deuterium plasma. In this note the emphasis is instead on the use of highly stripped high-Z (Z >> 2) ions, where Z-dependent effects are

correspondingly much larger. The work presented here was presented

informally by the author at the IAEA-sponsored "Technical Committee Meeting

on Mirror Fusion Research," Tsukuba University, Japan, July 8-12, 1985.

There are several physics issues attendant to the analysis of high-Z

minority ICRH. For the discussion here approximate treatments will be

given, adequate for estimating purposes; a more accurate analysis would

require the extensive use of Fokker-Planck and r.f. propagation codes,

involving detail beyond the intended scope of this note. The several issues

referred to, in the order that they will be discussed, are:

- Expected states of ionization (thus the cyclotron frequencies) of
  the high-Z ions.

- Power absorbtion by these ions from the r.f. electric field.

- Collisional coupling between the heated high-Z ions and the ions
  and electrons of the bulk plasma.

- Propagation of the r.f. wave into the plasma.

Ionic Charge States

Owing to the need for control of the type and the density of the high-Z

minority species ions that are to be r.f. heated it is proposed that they be

injected as neutral beams. Experience with nitrogen neutral beams on the

Livermore TMX-U experiment (unpublished work) has shown this to be a

practical proposition. The charge states reached by such injected neutrals

can be estimated by use of the so-called "coronal model" as extended to

fusion-relevant plasmas by, for example, Hulse, Post and Mikkelsen [5].

Examples of average equilibrium charge states, as deduced approximately by

interpolation from their analysis and in the limit of a vanishingly small

neutral hydrogen background are given in Table I. The examples represent

three candidate ions, nitrogen (Z = 7, A = 14), magnesium (Z = 12, A = 24),

and titanium (Z = 22, A = 48), evaluated at electron temperatures deemed

representative of present and possible future tandem mirror plasmas.

Table I.

Ion type	Electron temperature	Average charge state, $\bar{q}$
Nitrogen	250 eV	5
Magnesium	1 keV	10
Titanium	2 keV	20

In each example the distribution of charge states, though concentrated

at the average will necessarily be spread both above and below this value,

so that for nitrogen one would expect at least that the states q = 4, 5, and

6 would be present, and so forth. The presence of such spreads in charge

states implies a spread in cyclotron frequencies, but all lying below ω_{ci}

for the deuteron, by factors of 8/14 = 0.57 to 12/14 = 0.86 in the case of

nitrogen, factors of 0.75 to 0.92 for Mg. and 0.79 to 0.88 for Ti. This

circumstance suggests that the exciting r.f. frequency should be chosen so

that the highest expected high-Z ion's cyclotron frequency resonates near the minimum point between the central cell mirrors, with the adjacent, but lower, charged components then resonating partway up the positive magnetic gradient between the midplane and the mirrors. It follows that the r.f. frequency would then lie somewhat below the lowest value of the deuteron cyclotron frequency.

Alternatively, instead of relying on the magnetic field gradient to achieve resonances, multi-frequency r.f. excitation could be used, or single-frequency excitation resonating only with the dominant charge state of the minority high-Z ions, but still below ω_{ci} for deuterons in the vicinity of the antenna.

R.F. Power Absorption by the Minority Ions

In a paper analyzing the effects of fluctuations on plasma confinement in a tandem mirror, Rognlien and Matsuda [6] have derived expressions for the absorption of wave power centered near ω_{ci} for hydrogenic ions. It is a trivial task to extend their analytical results to arbitrary Z, A and then to employ the resulting expressions in estimating r.f. power absorption by high-Z ions.

The basic physics assumption implicit in Rognlien and Matsuda's analysis is that ions pass repeatedly through a "resonant surface" as they bounce between the mirrors, as a result of which they undergo stochastic heating, thereby absorbing energy from the r.f. electric field. Their expression (for the case of hydrogen ions) has been tested against a Monte

Carlo code and appears to be in good agreement, even at fairly high fluctuation levels, where the approximations involved might have been expected to fail. We therefore have some confidence in applying the extended expression to the present problem.

For heating near the fundamental resonance, ω_{ci}, and in the limit $k_\perp a_i \ll 1$ (a_i is the ion gyroradius), the extended expression for r.f. power absorbtion is:

$$P_{rf} = \frac{\pi q_i^2 e^2 E_{rf}^2 n_i L_B A_p}{4 M_i \omega_{ci}} \text{ watts} \quad . \tag{1}$$

Here n_i m^{-3} is the density of the ion species having a cyclotron frequency ω_{ci} rad sec^{-1}, ionic charge number q_i, and mass M_i kgm. L_B m is the scale length of the magnetic gradient in the vicinity of the resonant layer, and A_p m^2 is the plasma cross-sectional area, so that $N_i = n_i L_B A_p$ is simply the effective number of ions participating in the absorption. In the limit given ($k_\perp a_i \ll 1$) the power absorption is independent of the distribution function of the absorbing ions. Parenthetically, in the case of heating at the second harmonic, and for an assumed Maxwellian ion distribution function the above expression is found to be reduced by a factor $(k_\perp \bar{a}_i/2)^2$.

As can be seen from Eq. (1), P_{rf} scales as $(q_i^2/M_i \omega_{ci})$ i.e., as q_i, independent of M_i, favoring highly stripped icns of whatever mass.

An example, utilizing parameters appropriate to the Livermore TMX-U experiment [7], will illustrate the numbers involved. Assuming N^{5+} ions, $B = 0.4$ T, $L_B = 5$ m, and $r_p = 0.2$ m (in $A_p = \pi r_p^2$), one finds

$$P_{rf} = 9.8 \times 10^{-19} E_{rf}^2 n_i \text{ watts} \quad . \tag{2}$$

A typical value of E_{rf} for TMX-U, as calculated for 100 kW excitation of its antenna, using the McVey code [8], is 1500 V/m. At this r.f. field, Eq. (2) predicts an absorbed power of 100 kW at a N^{5+} density of only 4.5×10^{16} m^{-3}, i.e., of the order of one percent or less of the present operating deuterium plasma densities. This example illustrates the point earlier claimed, i.e., that only a small fractional component of high-Z ions is needed to accomplish transfer of the r.f. power to particle energy. For the number of participating ions assumed ($N_i = 2.8 \times 10^{16}$), even assuming a mean lifetime as short as 1 ms would only imply the need for a trapped N beam current of less than 5 amperes to maintain that number of N^{5+} ions.

Collisional Coupling to Hydrogenic Ions and Plasma Electrons

Collisional energy transfer rates between the r.f.-heated minority ions and the hydrogenic ions of the plasma may be estimated from simple collisional relaxation formulae, such as those given by Spitzer [9]. We have, for transfer of energy between energetic minority ions of charge

number q, density n_q m^{-3}, and energy W_q keV, and deuterons at density, n_D m^{-3}, subject to the requirement $W_q > (A_q/A_D) \overline{W}_D$:

$$\frac{dW_q}{dt} = - \frac{W_q}{\tau_{qD}} \quad , \tag{3}$$

with

$$n_D \tau_{qD} = 8.8 \times 10^{15} \left(1 + \frac{A_q}{A_D}\right)^{-1} (A_q^{1/2}/q^2) \; W_q^{3/2} \; m^{-3} \; sec \quad . \tag{4}$$

The power collisionally transferred is therefore

$$P_{qD} = n_q \frac{dW_q}{dt} \cdot V_p \quad , \tag{5}$$

with $V_p = L_p A_p$ being the plasma volume. From Eqs. (3), (4), and (5) there results

$$P_{qD} = 1.8 \times 10^{-32} W_q^{-1/2} \left(1 + \frac{A_q}{A_D}\right) (q^2/A_q^{1/2}) \; n_D n_q \; V_p \; watts \quad . \tag{6}$$

The scaling of transfer rates with charge and mass of the minority ion is seen to be as $q^2 (1 + A_q/A_D) A_q^{-1/2} = q^2 A_q^{1/2}$, a factor of 36 for N^{5+} ions

(relative to protons of the same energy), and a factor of 180 for Mg^{10+} ions. Furthermore, taking into account the dependence, as q, of power transfer from the r.f. field to the high-Z ions, it can be seen that each such ion is ~ $q^3 A_q^{1/2}$ times more effective in coupling r.f. energy to deuterons than would be a minority species of protons at the same energy.

It follows, as was the case with coupling to the r.f. wave, that effective collisional coupling of r.f.-derived energy from the high-Z minority ions requires only a small fractional density of these ions. For example, for TMX-U numbers, with L_p = 6 m and assuming N^{5+} as the minority ions one finds

$$P_{qD} \approx 0.73 \times 10^{-30} W_q^{-1/2} n_D n_q \quad . \tag{7}$$

For an operating deuteron ion density of 10^{19} m^{-3}, for example, and for, as before, an N^{5+} ion density of 4.5×10^{16} m^{-3} there results

$$P_{qD} \approx 3.3 \times 10^5 W_q^{-1/2} \quad , \tag{8}$$

so that if the N^{5+} ions are injected at a mean energy of about 10 keV their collisional transfer rate will be about 100 kilowatts, in this example roughly in equilibrium with the previously given r.f. heating power.

Alternatively, if the minority high-Z ions are introduced at low energy ($W_q < W_D$), they would be expected to be r.f. heated up to an energy $W_q > W_D$

such that the collisional power transfer to the deuterons again would just balance the r.f. power absorbed.

The simple examples given of course do not include the effects of energy absorption by electron drag, next to be discussed. This transfer rate can also be estimated from simple relaxation formulae, from which one finds

$$P_{qe} = 3.2 \times 10^{-34} W_q T_e^{-3/2} (q^2/A_q) \; n_q n_e V_p \; \text{watts} \quad . \tag{9}$$

The scaling here, as q^2/A_q, is not as strong with minority ion type as was the case for ion-ion transfer. For N^{5+} ions and TMX-U parameters one finds

$$P_{qe} = 4.3 \times 10^{-34} W_q T_e^{-3/2} \; n_q n_e \quad , \tag{10}$$

so that at $n_q = 4.5 \times 10^{16} \; m^{-3}$, $n_e = 10^{19} \; m^{-3}$,

$$P_{qe} = 190 \; W_q T_e^{-3/2} \quad . \tag{11}$$

At $W_q = 10$ keV and $T_e = 0.25$ keV, $P_{qe} = 15$ kilowatts. Thus the heating by collisional transfer from the minority ions to the electrons is seen to be substantially weaker in this example than the rate of transfer to plasma deuterons. However if these (the deuterons) are collisionally heated they

will in turn transfer energy by electron drag to the electrons. It is therefore probably not important that the direct collisional transfer from minority ions to electrons is relatively weak.

Comments on Propagation of the R.F. Wave

The fact that the cyclotron frequencies of the high-Z minority ions will generally lie below that of the deuterons of the plasma suggests that launching of the "fast wave" be essayed, for which $\alpha \equiv \omega_{cq}/\omega_{cD} < 1$. As discussed by Stix [10], in the slab approximation the propagation of the fast wave is governed by a dispersion relation of the form

$$N_{\perp}^2 = A - N^2 + A(1 - A)(A - N^2)^{-1} \quad , \tag{12}$$

with

$$N_{\perp,\parallel}^2 \equiv n_{x,z}^2 \ (B^2/4\pi n_i M_i c^2) \quad ;$$

$$n_{x,z} \equiv k_{x,z} \ c/\omega \quad ,$$

and

$$A \equiv \omega_{ci}^2/(\omega_{ci}^2 - \omega^2) = (1 - \alpha^2)^{-1} > 1 \quad .$$

One finds the requirement (for radial non-evanescence of the fast wave) that

$$0 \leq N_{\parallel}^2 \leq N_{\parallel 0}^2 = (1 + \alpha)^{-1} .$$
(13)

This result translates to the requirement that the majority species ion density should exceed a critical value:

$$n_D > 4.1 \times 10^{18} (1 + \alpha)/(\alpha^2 \lambda_z^2) \text{ m}^{-3} .$$
(14)

For $\lambda_z = 5$ m and $\alpha = 0.8$, for example, this implies $n_D > 4.6 \times 10^{17} \text{ m}^{-3}$. Since this density is lower than the normal operating density in TMX-U at least this requirement for radial non-evanescence ($N_{\perp}^2 > 0$) is in principle satisfied. Modal structure and mode conversion effects have here of course not been taken into account. The McVey code results cited earlier however indicate independently that both launching and penetration of the r.f. wave should be possible with a practical antenna structure.

Conclusions

The preceding discussion suggests that the absorption of r.f. power and its transfer to hydrogenic ions in a tandem mirror plasma may be enhanced if resonant excitation of a small constituent of neutral-beam-injected high-Z minority ions is essayed. This technique may also have applicability in other contexts, for example in a tokamak.

Acknowledgments

I would like to gratefully acknowledge the help of W. Cummins in performing the McVey code calculations and the useful discussions with D. Whang concerning r.f. propagation issues.

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